

# Grain size-dependent viscosity convection and the thermal evolution of the Earth

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## Abstract

Scaling relationships for grain size-dependent viscosity convection are derived assuming that eutectoid-like transformations at 660 km depth produce very small grains and that the grain size is mainly controlled by a coupled Ostwald ripening of Mg-perovskite, Ca-perovskite and magnesiowüstite. Parameterized convection calculations based on these relationships show that the thermal evolution of the Earth strongly depends on the grain growth parameters. The Earth can either quickly forget the initial conditions with temperature and heat loss following the decaying radiogenic heat production (Tozer-type evolution) or the initial conditions can essentially determine the temperature and heat loss (Christensen-type evolution). Christensen-type models can easily satisfy both the present-day heat flux and geochemical constraints on abundances of radiogenic isotopes. For example, this happens if the rate-controlling process for both grain growth and viscous creep is volume diffusion and the activation enthalpy for grain growth is about 50% higher than that for viscous creep. © 2001 Elsevier Science B.V. All rights reserved.

*Keywords:* grain size; viscosity; phase transitions; mantle; convection; thermal evolution

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## 1. Introduction

The current views on the thermal evolution of the Earth have been substantially influenced by Tozer's [1] idea that a strong dependence of viscosity on temperature implies that only small variations in temperature are needed to accommodate the gradual decay of radiogenic heat production during planetary evolution and that the heat loss of the planet must closely follow

the heat production rate. Early models of the thermal evolution of the Earth found that the present-day temperature is indeed only a hundred degrees lower than that in the Archean and that the present-day heat loss rate exceeds the heat generation rate by about 10% [2]. The excess heat loss rate is due to secular cooling [2]. A higher contribution from secular cooling obtained in other studies is due to substitution of the radioactive decay law for the four major isotopes by a single exponential function with some effective half-decay time (see discussion in [3]). It was also found that the thermal evolution does not depend on the initial temperatures except for the first  $5 \times 10^8$  yr (the thermal adjustment time). Model temperatures in the Archean are generally

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consistent with estimates of mantle temperature based on liquidus of high-magnesian komatiites. The uncertainties are largely due to the possibility that komatiites might represent temperatures of hot plumes rather than average mantle [4,5] and that formation of komatiites depends not only on temperature, but also the abundance of water in the mantle [6].

A major difficulty was to obtain the observed present-day heat flux. The models required that the abundances of radiogenic isotopes must be significantly higher than those suggested by geochemical models [7] (at least by 50%). Further studies discovered a variety of factors which can affect the thermal evolution of the Earth and reconcile geochemical and geodynamical models. In particular, it was found that the heat flux, the thermal adjustment time and the temperature drop since the Archean are strongly affected by layering [8]. Although the original model of layered convection was difficult to reconcile with tomographic evidence for whole-mantle circulation [9,10], secular variations in the degree of layering due to phase transformations [11–13] offered a possible compromise. Continents [14], partial melting [15], and gradual differentiation of radiogenic elements into the crust [16] were also shown to be important factors.

A very different class of models was proposed by Christensen [3,17] who looked at the effect of rheology. He found that a sluggish, high-viscosity lithosphere can weaken the feedback between the heat flux and the interior viscosity. This increases both the heat flux and the thermal adjustment time of the planet. Although subsequent studies showed that weak zones [18] reduce the role of high-viscosity lithosphere and that Christensen's parameter range is in the transitional regime of temperature-dependent viscosity convection [19], the models demonstrated that planetary evolution can depend on the rheology of the mantle and lithosphere. Detailed studies of convection with strongly temperature-dependent viscosity showed that the role of rheology is so crucial that it might determine the presence or absence of plate tectonics on a planet [20–22].

So far, only a few aspects of variable viscosity convection have been understood in the context

of planetary evolution. These include temperature-dependent viscosity, power-law creep and yield stress. The importance of other factors remain unknown. This study explores the role of grain size-dependent viscosity.

## 2. Grain size-dependent viscosity

Convective flow in the lower mantle of the Earth is likely to occur in the diffusion or superplastic creep [23]. In either case, the viscosity of the lower mantle is a strong function of the grain size. In steady-state ductile creep, it can be written as: ([24])

$$\eta \propto d^p \tau^{1-n} \exp \frac{Q}{RT} \quad (1)$$

where  $T$  is the temperature,  $\tau$  is the second invariant of the deviatoric stress tensor,  $Q = E^* + PV^*$  is the activation enthalpy,  $P$  is the hydrostatic pressure,  $R$  is the gas constant,  $E^*$  is the activation energy,  $V^*$  is the activation volume,  $d$  is the grain size,  $n$  is the stress exponent, and  $p$  is the grain size exponent. It is convenient to use strain rate  $\dot{\epsilon} = \tau/\eta$  rather than stress:

$$\eta \propto d^{p/n} \dot{\epsilon}^{(1-n)/n} \exp \frac{Q}{nRT} \quad (2)$$

Although the constraints on the viscosity of the lower mantle are poor, it is possible to put some brackets on the parameter range. Firstly, the absolute value of the viscosity, and thus the pre-factor (which is omitted in the above equation), are not important for scaling relationships. The three key parameters which are critical for the thermal evolution of the Earth are  $Q$ ,  $n$  and  $p$ .

Secondly, we will assume that (Mg,Fe)-perovskite, which occupies about 75% by volume, controls the viscosity of the lower mantle. In principle, the presence of magnesiowüstite could substantially reduce the viscosity of the lower mantle if it formed continuous films around grains of perovskite [32]. However, formation of such films requires more than 250% shear strain (see [32] and references therein), while the strain in the lower mantle is only 50% or so. Besides, the grain size in the lower mantle seems to be small

enough for the surface tension to be larger than the viscous stress [31]. In such a case, the grains of magnesiowüstite would preserve a nearly equiaxial shape during Ostwald ripening (see below) and remain isolated from each other.

The value of  $Q$  (a depth average) for (Mg,Fe)-perovskite could be as low as  $300 \text{ kJ mol}^{-1}$  or as high as  $1500 \text{ kJ mol}^{-1}$  depending on the poorly constrained activation parameters, oxygen fugacity and other factors [25–29]. The grain size exponent,  $p$ , is typically between 2 and 3, and the stress exponent,  $n$ , is between 1 and 2 ( $n=2$  is sometimes observed in the superplasticity regime [30]).

It is difficult to reduce these uncertainties further. Nevertheless, there are a few important observations. Experiments on an analogue mineral,  $\text{CaTiO}_3$ , show that for grain sizes between 5 and  $60 \mu\text{m}$ ,  $n \approx 1$  and  $p \approx 2$  [26]. The deformation mechanism was probably diffusion creep controlled by volume diffusion, although grain-boundary sliding was also present as indicated by nearly equiaxed grains (a signature of superplasticity). Estimates of the grain growth rates in the lower mantle show that if silicon is the rate-controlling species, then the grain size is about  $100\text{--}1000 \mu\text{m}$  and that volume diffusion is likely to be the rate-controlling process [31]. Even if superplasticity was partially present in [26], at  $100\text{--}1000 \mu\text{m}$  grain size superplasticity is likely to be completely replaced by diffusion creep as it happens in most materials at such a relatively large grain size (grain size is one of the parameters which controls the transition from superplasticity to diffusion creep [30]). Although robust constraints can only be obtained in future laboratory experiments, at present, diffusion creep (and more specifically, volume diffusion) appear to be the most likely creep mechanism in the lower mantle. Therefore, the ‘preferable values’ of the exponents in the viscosity law are  $n=1$  and  $p=2$ .

### 3. Grain size

The grain size is determined by dynamic recrystallization, phase transformations and grain growth. Dynamic recrystallization is believed to

take place only in the dislocation creep regime. In the diffusion creep or superplastic regime, the grains do not reach the critical size at which grain size reduction processes (such as nucleation of new grains and subgrain rotation) become efficient. Therefore, the grain size in the lower mantle is determined by phase transformations and grain growth.

The eutectoid decomposition of spinel to perovskite and magnesiowüstite at 660 km produces very thin lamellae of alternating perovskite and magnesiowüstite with the spacing  $0.1\text{--}1 \mu\text{m}$  [29,33–37]. The grain size after spheroidization of the lamellar structure is probably about  $1 \mu\text{m}$  [31]. The kinetics of phase transformations in the pyroxene subsystem is not well understood [38]. Since it produces two phases, a fine lamellar structure and small initial grains can be expected in this subsystem as well.

How fast do grains grow after the phase transformations? According to [31], the initial grain growth is due to grain boundary migration in (Mg,Fe)-perovskite phases. Eventually this period of very fast growth ceases due to the presence of magnesiowüstite and Ca-perovskite phases (Zener pinning). The grain size of (Mg,Fe)-perovskite reaches a maximum value which is a factor of two larger than the size of the second phase particles. Further growth of (Mg,Fe)-perovskite grains is determined by Ostwald ripening of the second phase particles (Fig. 1). In the presence of several minor phases (magnesiowüstite and Ca-perovskite), the slowest growing phase is likely to control the grain growth rate of all other phases.

Ostwald ripening is a thermally activated process and is usually described with the help of a power-law function of time:

$$d \propto \left[ t \exp \left( -\frac{Q_{\text{gr}}}{RT} \right) \right]^{1/m} \quad (3)$$

where  $m$  is the grain growth exponent and  $Q_{\text{gr}}$  is the activation enthalpy for grain growth. The value of  $m$  depends on whether Ostwald ripening is controlled by volume diffusion ( $m=3$ ), grain-boundary diffusion ( $m=4$ ) or diffusion on dislocations ( $m=5$ ).

## Grain growth in the lower mantle

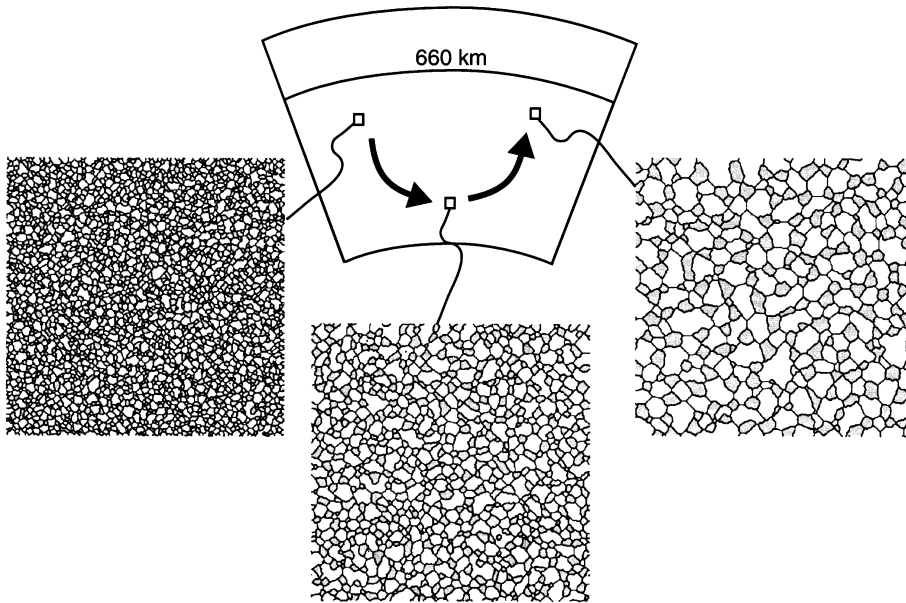


Fig. 1. Schematic illustration of grain growth in the lower mantle. The microstructures are from actual numerical simulations for a two-phase system consisting of 70% major phase and 30% minor phase [31].

Yamazaki et al. [29] found that  $m \sim 10\text{--}11$  for the perovskite–periclase system. However, such a high value of  $m$  is difficult to reconcile with asymptotic grain growth theories, but can be explained by transient grain growth due to coarsening and spheroidization of the lamellar structure and elastic coupling between the grains [31]. This implies that on convective time scales, the grain growth is expected to be in one of the asymptotic regimes of Ostwald ripening with  $m$  between 3 and 5.

To estimate  $m$  and  $Q_{\text{gr}}$ , we need detailed knowledge of the diffusivities, solubilities and partition coefficients of all major and minor elements (since Ostwald ripening is driven by tiny differences in concentrations, a minor element with a very low diffusivity and which partitions mainly to magnesiowüstite and Ca-perovskite might become the rate-limiting species [31]). In particular, this implies that the rate-controlling species for grain growth can be different from the rate-controlling species for viscous creep. Therefore, there could

be no correlation between  $m$  and  $p$  or between  $Q$  and  $Q_{\text{gr}}$ .

A substantially simpler situation can take place if the same species is the rate-controlling species for both grain growth and creep. In this case, some correlation between the grain growth parameters and the viscosity parameters can be expected. For instance, if silicon is such a rate-controlling diffusion species [39], then both grain growth and viscous creep are likely to be controlled by the same diffusion mechanism, with respect to where diffusion occurs, on the grain boundaries or in the lattice. This can be seen from the fact that both the transition from grain–boundary diffusion controlled growth to volume diffusion-controlled growth and the transition from grain boundary diffusion controlled creep to volume diffusion controlled creep occurs when the grain size exceeds  $\sim \delta D_{\text{GB}}/D_{\text{V}}$ , where  $\delta$  is the width of grain boundaries,  $D_{\text{GB}}$  is the coefficient of grain–boundary diffusion and  $D_{\text{V}}$  is the coefficient of volume diffusion. Therefore,

the values of  $m$  and  $p$  are likely to correlate: in the grain-boundary diffusion regime  $m=4$  and  $p=3$ ; in the volume diffusion regime  $m=3$  and  $p=2$ . The activation enthalpies for grain growth and viscous creep would also be similar. The choice can further be narrowed down if silicon is indeed the rate-controlling species. The data on the diffusion of silicon [39] suggest that grain growth is likely to be controlled by volume diffusion. This gives ‘preferable parameters’ for grain growth:  $m=3$  and  $Q_{\text{gr}} \approx Q$ . The value of  $m=3$  implies that  $p=2$ , which is in agreement with our earlier choice of preferable parameters for viscous creep.

#### 4. Scaling

Before deriving scaling relationships, it might be worth emphasizing that whenever average properties of a convective layer are considered, the largest concern is spatial variations of the viscosity. Previous studies showed that average properties can be used for temperature-dependent viscosity convection with small viscosity contrasts ( $<100$ ) [19] and power-law viscosity convection where the viscosity varies by many orders of magnitude due to stress variations [19,40]. Pressure-dependent viscosity can also be described by appropriately chosen average parameters [41]. In the extreme cases when large viscosity variations between the surface and the interior completely immobilize the surface, convection beneath the stagnant lid can still be described with the help of average parameters in the actively convective region [19]. Even in rather complex situations where a yield stress is used to mobilize the stagnant lid and initiate subduction, average parameters of the interior region still represent the convection system despite large viscosity contrasts between the plate and interior [22]. Therefore, previous studies of convection with complex rheologies suggest that average parameters can probably be used for grain size-dependent viscosity convection as well.

Although the scaling relationships for grain size-dependent viscosity convection can be derived following a standard approach [19], a much sim-

pler approach is to show that grain size-dependent viscosity scales the same way as power-law viscosity with some apparent parameters. Since the scaling laws for power-law viscosity are known, the scaling laws for grain size-dependent viscosity will follow immediately.

The viscosity of the lower-mantle scales as:

$$\eta \propto d^{p/n} \dot{\epsilon}^{(1-n)/n} \exp \frac{Q}{nRT} \quad (4)$$

where  $\bar{T}$  is the average temperature of the lower mantle (which is approximately the same as the average temperature of the entire mantle).

The characteristic strain rate is:

$$\dot{\epsilon} \sim u/l \quad (5)$$

where  $l$  is the travel distance in the lower mantle (it approximately scales with the depth of the entire mantle) and  $u$  is the convective velocity.

The characteristic grain size is:

$$d \propto t_{\text{gr}}^{1/m} \exp -\frac{Q_{\text{gr}}}{mRT} \quad (6)$$

where the time available for grain growth is the residence time of grains in the lower mantle:

$$t_{\text{gr}} \sim l/u \quad (7)$$

This scaling is only valid for whole-mantle convection. If mantles were separated into two convective layers at 660 km depth, the material would not cross the phase boundary, the phase transformation induced recrystallization at 660 km depth would not take place, and the characteristic time available for grain growth in the lower mantle would be the evolution time.

The scaling law for the viscosity of the lower mantle becomes:

$$\eta \propto u^{(1-n)/n-p/nm} \exp \frac{(Q-pQ_{\text{gr}}/m)}{nRT} \quad (8)$$

This is exactly the same scaling law which describes a power-law viscosity without grain size

dependence (i.e.,  $p=0$  in Eq. 4). It can be rewritten as:

$$\eta \propto u^{(1-n')/n'} \exp \frac{Q'}{n'RT} \quad (9)$$

where the apparent  $n'$  and  $Q'$  are defined by the following equations:

$$\frac{1-n}{n} - \frac{p}{nm} = \frac{1-n'}{n'} \quad (10)$$

and

$$\frac{Q}{n} - \frac{pQ_{gr}}{nm} = \frac{Q'}{n'} \quad (11)$$

Thus:

$$n' = \frac{nm}{m-p} \quad (12)$$

and

$$Q' = \frac{mQ - pQ_{gr}}{m-p} \quad (13)$$

This shows that grain size-dependent viscosity can approximately be interpreted as a non-Newtonian viscosity with some apparent stress exponent and activation enthalpy.

This apparent non-Newtonian viscosity can be interpreted further in terms of Newtonian viscosity with a reduced activation enthalpy [42]:

$$n'' = 1 \quad (14)$$

$$Q'' = \frac{3}{n'+2} Q' = \frac{3(mQ - pQ_{gr})}{(n+2)m - 2p} \quad (15)$$

For  $n=1$ , Eq. 15 is identical to Eq. 11 from [43]. Although this is a less accurate interpretation and will not be used below, it does capture the role of grain size-dependent viscosity in a single formula.

Scaling laws for convection with the viscosity described by Eq. 9 have been derived in [19]. With realistic parameters, the viscosity contrasts are usually so large that convection occurs not in the plate tectonics regime, but in the stagnant lid regime. However, a small yield stress for brittle

failure (which is probably due to water) remobilizes the surface layers and brings the convective style to a regime with mobile plates [22]. In this regime the heat flux depends on temperature as [19]:

$$F(T_m) \propto \Delta T^{2(n'+1)/(n'+2)} \exp \left[ -\frac{Q'}{(n'+2)R\bar{T}} \right] \quad (16)$$

where  $\Delta T = T_m - T_s$ ,  $T_s$  is the surface temperature,  $T_m$  is the mantle temperature beneath the lithosphere and the average temperature  $\bar{T}$  of an adiabatic mantle is related to  $T_m$  via a simple relationship:

$$\bar{T} = \zeta T_m \quad (17)$$

where the constant  $\zeta$  is about 1.3. The major uncertainty in the above relationship is associated with pressure-dependent viscosity. Scaling laws for convection with pressure-dependent viscosity are not well understood for the plate tectonics regime. It seems reasonable to assume that if plates penetrate through the lower mantle, which is the largest and the most viscous part of the mantle, then the activation parameters, as well as the effective temperature (which can be approximated by the average mantle temperature) in Eq. 16, must be calculated at the lower-mantle conditions.

## 5. Parameterized convection

The consequences of grain size-dependent viscosity convection for the thermal evolution of the Earth can be understood with the help of parameterized convection models [2,3]. No attempt is made here to construct a complete model of thermal evolution. Instead, the simplest possible model is developed to emphasize the consequences of grain size-dependent viscosity.

The energy balance for the mantle, ignoring the heat flux from the core, can be written as:

$$\zeta C_p M \frac{dT_m}{dt} = -F(T_m)S + MH(t) \quad (18)$$

where  $C_p$  is the specific heat,  $M$  is the mass of the

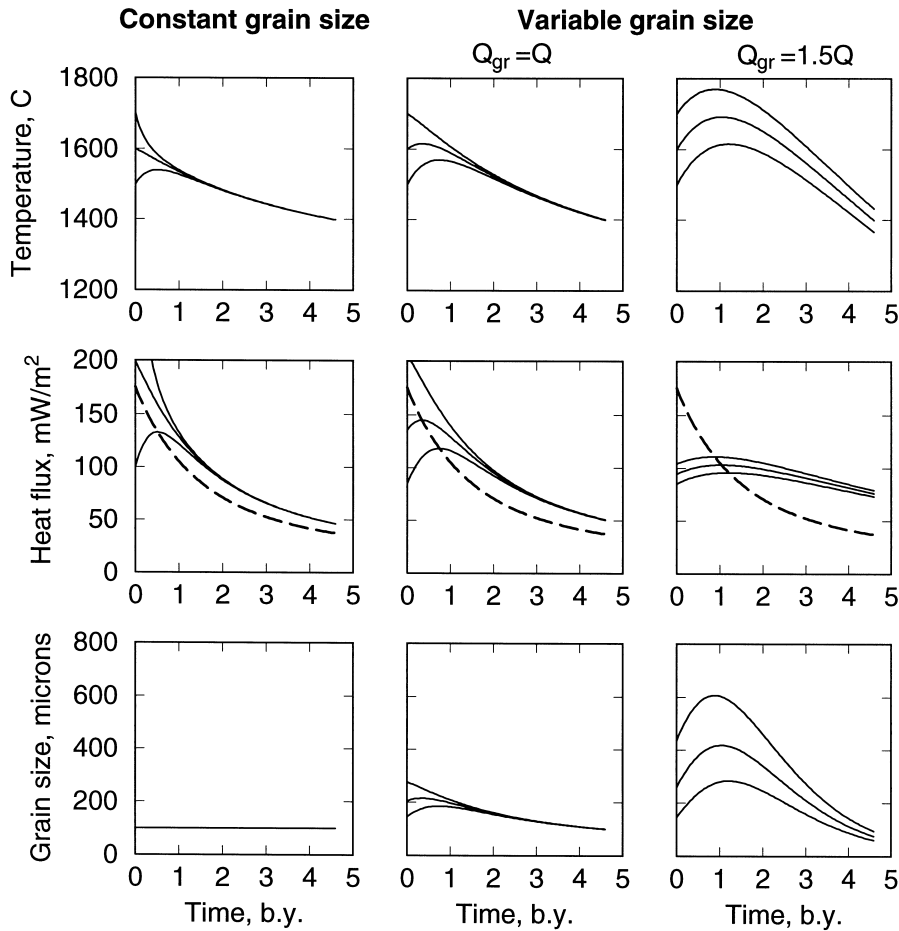


Fig. 2. Parameterized convection calculations of the thermal evolution of the Earth. The constant grain size model assumes that the viscosity only depends on temperature. The grain size-dependent viscosity model assumes  $n=1$ ,  $p=2$ , and  $m=3$ . The results are shown for two different ratios of  $Q_{gr}/Q$ : 1 and 1.5. The three curves on each graph correspond to different initial temperatures, 1500, 1600 and 1700°C. The models are constrained to give the same present-day mantle temperature ( $\sim 1400^\circ\text{C}$ ) and the same grain size ( $\sim 100\ \mu\text{m}$ ). For  $Q_{gr}/Q=1.5$  these constraints are satisfied only approximately because the present-day thermal state depends on the initial conditions.

mantle,  $S$  is the surface area of the Earth,  $F(T_m)$  is the surface heat flux, and  $H(t)$  is the radiogenic heat production rate per unit mass.

Numerical results are shown in Figs. 2 and 3. The parameters used in these calculations are  $m=3$ ,  $p=2$ ,  $n=1$ ,  $Q/\zeta=500\ \text{kJ mol}^{-1}$ ,  $M=4.1 \times 10^{24}\ \text{kg}$ ,  $S=5.1 \times 10^{14}\ \text{m}^2$ ,  $C_p=1200\ \text{J kg}^{-1}\ \text{K}^{-1}$ ,  $\Delta T=1400\ \text{K}$ , and  $T_m=1673\ \text{K}$ . The internal heat production is due to radioactive decay of four major isotopes:  $^{235}\text{U}$ ,  $^{238}\text{U}$ ,  $^{232}\text{Th}$  and  $^{40}\text{K}$ . We assume  $\text{K}/\text{U}=10^4$  and

$\text{Th}/\text{U}=3.8$ , and that the abundance of  $\text{U}$  is close to the chondritic value of 20 ppb [7].

The effect of grain size-dependent viscosity convection is small for small values of the activation enthalpy  $Q_{gr}$  for grain growth. When  $Q_{gr} \approx Q$ , the cooling rate of the Earth is about 50% higher than for a model with a constant grain size, and the total heat flux is about  $50\ \text{mW m}^{-2}$ . This is still smaller than the observed surface heat flux which is  $86\ \text{mW m}^{-2}$  [44] or about  $75\ \text{mW m}^{-2}$  if we assume that about  $11\ \text{mW m}^{-2}$  is due to the heat

flux from the core [45–47]. As  $Q_{\text{gr}}$  increases, the cooling rate increases substantially and the surface heat flux can easily reach the required values (Fig. 3).

These results can be understood with the help of a simple asymptotic analysis. If the cooling rate is very slow the equation:

$$0 = -F(T_{\text{m},0})S + MH(t) \quad (19)$$

gives an approximate, asymptotic solution for  $T_{\text{m},0}(t)$ , which does not depend on the initial conditions.

The surface heat flux can approximately be calculated as:

$$F = F_H + \Delta F \quad (20)$$

where:

$$F_H = MH(t)/S \quad (21)$$

is a quasi-equilibrium heat flux,  $\Delta F$  is the excess heat flux due to secular cooling:

$$\Delta F = \frac{C_p M}{S} \frac{dT_{\text{m},0}}{dt} = \frac{C_p M \Delta T (n' + 2)}{St_H (\theta' + 2n' + 2)} \quad (22)$$

$$\theta' = \frac{Q' \Delta T}{RT_{\text{m},0}^2} \quad (23)$$

is the apparent Frank–Kamenetskii parameter and  $t_H = (d \ln H/dt)^{-1}$ .

These formulas agree very well with the numerical solutions (Fig. 3) except for the high heat flux region, where the asymptotic analysis is inaccurate. They show that larger  $Q_{\text{gr}}$  corresponds to smaller  $Q'$  and  $\theta'$  and larger heat flux. At lower values of  $Q_{\text{gr}}$ , the heat flux  $\Delta F$ , due to secular cooling, is small and the heat flux approaches its quasi-equilibrium value  $F_H$ , Eq. 21.

The thermal adjustment time (during which, the initial temperatures are forgotten) is related to  $\Delta F$  via a general formula [48]:

$$t_{\text{th}} = t_H \frac{\Delta F}{F_H} \quad (24)$$

## Present-day heat flux

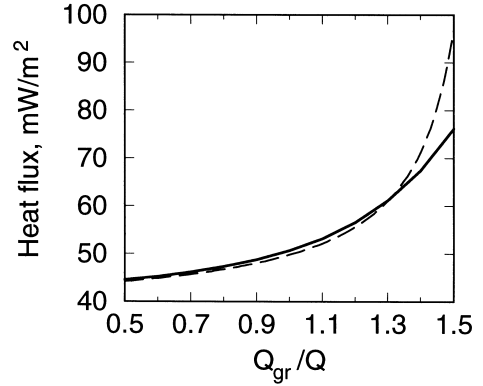


Fig. 3. The present-day heat flux as a function of the ratio of the activation enthalpy  $Q_{\text{gr}}$  for grain growth to the activation enthalpy  $Q$  for viscous creep. The grain growth function and the viscosity function are the same as in Fig. 2. All models give the same present-day mantle temperature (1400°C). The asymptotic solution, Eqs. 20–22, is shown with a dashed line.

This shows that the initial conditions would affect the thermal state of the present-day Earth ( $t_{\text{th}} > 4.6 \times 10^9$  yr) if  $\Delta F > 0.8F_H$  (the present-day value of  $t_H$  is about 5.7 000 000 000 yr). This explains why the grain size-dependent viscosity model with  $Q_{\text{gr}} = 1.5Q$  (Fig. 2) depends strongly on the initial conditions.

It is worth noting that if the grain growth exponent,  $m$ , was indeed about 10 or 11 as suggested in [29], the effect of grain growth on thermal evolution would be negligible. The deviation of the apparent stress exponent and the apparent activation enthalpy from their ‘normal’ values for diffusion creep, the change in the secular cooling rate, Eq. 22, and the change in the thermal adjustment time, Eq. 24, would only be about 10 to 20%. Interestingly enough, at even larger values of  $m$  (when the grains would barely be able to double their initial size) the initial grain size established after the phase transformations would play the crucial role [43]. The scaling relationships would need to be modified accordingly. At present, there is little support for such abnormally large values of  $m$ .

## 6. Conclusion

The role of grain size-dependent viscosity convection in the thermal evolution of the Earth can be understood through interpreting grain size-dependent viscosity convection in terms of power-law viscosity convection with an apparent stress exponent and activation enthalpy.

The thermal evolution of the Earth strongly depends on the physical laws controlling grain growth in the lower mantle. If both grain growth and diffusion creep are controlled by volume diffusion of the same species and have similar activation enthalpies (that is stress exponent  $n=1$ , grain size exponent  $p=2$ , grain growth exponent  $m=3$  and  $Q_{gr} \approx Q$ ) the thermal evolution of the Earth is qualitatively similar to previously suggested models (Tozer-type evolution) but with a higher present-day cooling rate of the Earth.

Deviations from this parameter set can cause dramatic changes in planetary evolution: the thermal evolution can depend on the initial conditions and the heat flux can be much higher than the equilibrium one corresponding to the radiogenic heat production (Christensen-type evolution). For example, the present-day surface heat flux can easily be reconciled with chondritic abundances of radiogenic elements provided the activation enthalpy  $Q_{gr}$  for grain growth is 50% higher than the activation enthalpy  $Q$  for diffusion creep.

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