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Physics of the Earth and Planetary Interiors 116 (1999) 1–7

PHYSICS
OF THE EARTH
AND PLANETARY
INTERIORS

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Stagnant lid convection in a spherical shell

C.C. Reese [a,*], V.S. Solomatov [a,1], J.R. Baumgardner [b,2], W.-S. Yang [c,3]

[a] Department of Physics, New Mexico State University, Las Cruces, NM 88003, USA
[b] Theoretical Division, MS B216, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
[c] Institute of Geophysics and Planetary Physics, MS C305, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

Received 23 March 1999; accepted 14 May 1999

Abstract

A compelling explanation for the observed lack of mobile surface plates on the terrestrial planets is that the strong temperature dependence of silicate rheology leads to a quasi-rigid stagnant lid at the cool surface of the convecting planetary mantle. We investigate such stagnant lid convection in an internally heated spherical shell. For the parameter range we study, convection beneath the lid is steady in time and characterized by cylindrical upwellings surrounded by cold sheet-like downwellings that exhibit dodecahedronal ($l = 6$, $m = \{0,5\}$) symmetry. The scaling relationship we obtain for the heat flux is very similar to results from two-dimensional numerical experiments and asymptotic boundary layer analyses. It seems that the predictions of two-dimensional models, in particular, low heat transport efficiency and extensive melting, also apply in fully three-dimensional spherical geometry. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Stagnant lid convection; Planetary heat flow; Mantle rheology

1. Introduction

Mobile lithospheric plates that themselves participate in convective mantle circulation seem to be unique to the Earth. By contrast, the tectonic styles of the other terrestrial planets appear to reflect the stagnant lid regime of mantle convection in which the cold upper thermal boundary layer becomes stiff and immobile because of its high viscosity (Nataf and Richter, 1982; Morris and Canright, 1984; Fowler, 1985; Davaille and Jaupart, 1993; Solomatov and Moresi, 1996). Thermal convection with relatively mild ($\sim 10^3$) temperature induced viscosity variation has been investigated numerically in three-dimensional Cartesian (Christensen and Harder, 1991; Tackley, 1993, 1996) and spherical shell (Ratcliff et al., 1995, 1996b) geometries. The stagnant lid regime, which occurs at viscosity contrasts of about 10^4 (Solomatov, 1995), was reached in three-dimensional simulations by Ogawa et al. (1991), Trompert and Hansen (1998) and Ratcliff et al. (1996a; 1997).

One of the implications of stagnant lid convection is its extremely low efficiency of heat transfer to the surface (Schubert et al., 1997; Reese et al., 1998). While spherical geometry is most appropriate for the mantles of terrestrial planets, existing parameterizations of stagnant lid heat flux are limited by the

* Corresponding author. Fax: +1-505-646-1934; e-mail: creese@nmsu.edu
[1] E-mail: slava@nmsu.edu.
[2] E-mail: johnrb@lanl.gov.
[3] E-mail: wyang@lanl.gov.

PII: S0031-9201(99)00115-6

assumption of Cartesian geometry. We investigate stagnant lid mantle convection with strongly temperature dependent viscosity in a spherical shell with purely internal heating (the primary mode of heating in planetary mantles) and compare scaling relations for the heat transfer with the results from two-dimensional parameterizations.

2. Parameters

The viscosity of rocks has an Arrhenius temperature dependence which for diffusion creep can be written as (Karato and Wu, 1993):

$$\eta = b \exp\left(\frac{Q}{RT}\right) \tag{1}$$

where b is a constant, Q is the activation enthalpy, R is the gas constant, and T is the temperature. For the purposes of this study, we make the Frank–Kamenetskii approximation:

$$\eta = b' \exp(-\gamma T), \tag{2}$$

where b' and γ are constants that depend on the temperature of the isothermal interior,

$$\gamma = Q/RT_{\mathrm{i}}^2. \tag{3}$$

The nature of this approximation is that at large viscosity contrasts, dynamically important processes only occur in the approximately isothermal interior. The large differences between Eqs. (1) and (2) near the surface of the lid are unimportant for the heat transfer.

The problem is specified in terms of two parameters: the rheological gradient:

$$\gamma = -\mathrm{d}\ln\eta/\mathrm{d}T, \tag{4}$$

and the surface Rayleigh number:

$$Ra_{H,0} = \frac{\alpha\rho^2 gHd^5}{k\kappa b' \exp(-\gamma T_{\mathrm{s}})}, \tag{5}$$

where T_{s} is the surface temperature, α is the coefficient of thermal expansion, ρ is the density, g is the gravitational acceleration, H is the internal heating rate, $d = r_{\mathrm{t}} - r_{\mathrm{b}}$ is the shell thickness, $\kappa = k/\rho c_{\mathrm{p}}$ is the thermal diffusivity, k is the thermal conductivity, c_{p} is the specific heat capacity at constant pressure. The internal temperature T_{i}, defined as the mean temperature at the inner boundary of the spherical shell, is determined by numerical experiment.

The Nusselt number for internal heating is expressed as the ratio of the conductive temperature contrast across the shell to the actual temperature contrast across the shell:

$$Nu = \frac{\rho H r_{\mathrm{t}}^2 \left(1 - 3r_{\mathrm{b}}^2/r_{\mathrm{t}}^2 + 2r_{\mathrm{b}}^3/r_{\mathrm{t}}^3\right)}{6k(T_{\mathrm{i}} - T_{\mathrm{s}})}. \tag{6}$$

The Frank–Kamenetskii parameter based on the heating rate:

$$\theta = \frac{\rho H r_{\mathrm{t}}^2 \left(1 - 3r_{\mathrm{b}}^2/r_{\mathrm{t}}^2 + 2r_{\mathrm{b}}^3/r_{\mathrm{t}}^3\right)}{6k}\gamma. \tag{7}$$

The physical significance of θ is made clear by the expression for the viscosity contrast across the shell

$$\Delta\eta = \exp(\theta/Nu). \tag{8}$$

3. Numerical simulations

For our study of thermal convection in a spherical shell, we choose free slip boundary conditions and an inner to outer radius ratio $r_{\mathrm{b}}/r_{\mathrm{t}} = 0.55$ which is characteristic of the mantles of the terrestrial planets.

The numerical code TERRA (Baumgardner, 1985) modified for variable viscosity (Yang, 1997) utilizes a computational mesh based on the regular icosahedron which yields an almost uniform discretization of the sphere (Baumgardner and Fredrickson, 1985). A convergence test for $Ra_{H,0} = 2.5 \times 10^3$, $\theta = 8.2$ gives $Nu = (2.905, 2.894, 2.884)$ for the number of radial layers 8, 16 and 32, respectively. The asymptotic extrapolated value is about $Nu = 2.863$ so the 16 radial layer grid gives an accuracy of $\sim 1\%$.

Six steady state cases were run on a 16 radial layer mesh (Table 1). Convection beneath the lid is

Table 1
Numerical results

$Ra_{H,0}$	θ	Nu	$Ra_{H,i}$ $(\times 10^5)$[a]	$\Delta\eta$ $(\times 10^5)$
3	14	1.20	3.51	1.17
10	14	1.31	4.40	0.44
30	14	1.42	5.70	0.19
0.3	17.5	1.20	6.48	21.6
1	17.5	1.28	8.66	8.66
3	17.5	1.36	11.6	3.88

[a] $Ra_{H,i} = Ra_{H,0}\Delta\eta$.

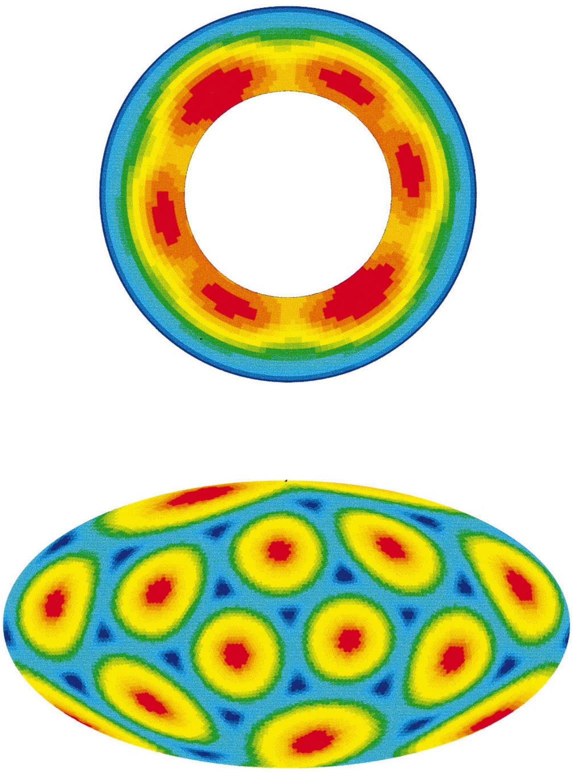

Fig. 1. Pattern of convection for the case $Ra_{H,0} = 10$, $\theta = 14$. Equatorial (top) and mid-depth equal area projection (bottom) of the temperature field. Blue represents cold fluid and red represents hot fluid. The coldest portion of the shell (blue–light blue in the top figure) is stagnant and does not participate in convection.

characterized by quasi-cylindrical upwellings surrounded by cold sheet-like downwellings. The horizontal planform of the flow has Y_6^5 and Y_6^0 components where Y_l^m is the spherical harmonic of degree l and order m (Fig. 1).

4. Asymptotic heat flux scaling

Of particular importance for mantle dynamics and evolution is the relationship between the surface heat flux and the interior temperature. In this section, we develop a model for the low Nusselt number mantle heat flux and fit the model parameters using the numerical data. This expression is then extrapolated to the asymptotic thin lid case which is more applicable to the mantles of the terrestrial planets.

For spherical shell geometry, the heat flux:

$$F = \frac{1}{3}\rho H r_t \left(1 - r_b^3/r_t^3\right), \tag{9}$$

which can be written in terms of the Nusselt number (6):

$$F = \frac{k(T_i - T_s)}{d}\,\frac{2\left(1 - r_b^3/r_t^3\right)\left(1 - r_b/r_t\right)}{\left(1 - 3r_b^2/r_t^2 + 2r_b^3/r_t^3\right)} Nu. \tag{10}$$

In the numerical simulations, the lid thickness is approximately $1/4$ of the shell thickness (Fig. 2). To take into account the fact that heat producing elements in the stagnant lid do not drive convection in

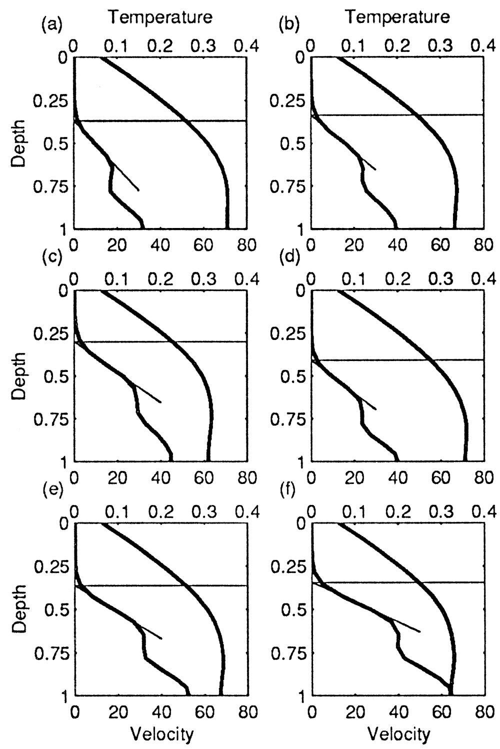

Fig. 2. The radial distribution of horizontally averaged velocity and temperature for the cases $(Ra_{H,0}, \theta) = (3,14)$, $(10,14)$, $(30,14)$, $(0.3,17.5)$, $(1,17.5)$, $(3,17.5) =$ a,b,c,d,e,f. The base of the stagnant lid (horizontal line) is defined by the interior velocity gradient. This definition gives $a_{rh} \approx 3.7 \pm 0.6$. The velocity and temperature scales are κ/d and $\rho H d^2/k$, respectively.

the mantle, we write the effective heat flux beneath the lid as:

$$F_{\text{eff}} = \frac{1}{3}\rho H r_1\left(1 - r_b^3/r_1^3\right). \tag{11}$$

Additionally, we assume that convection beneath the lid is similar to constant viscosity convection with an effective driving temperature γ^{-1} (Grasset and Parmentier, 1998; Reese et al., 1999):

$$F_{\text{eff}} = a\frac{k\gamma^{-1}}{d_{\text{eff}}}\left(\frac{\alpha\rho g\gamma^{-1}d_{\text{eff}}^3}{b'\kappa\exp(-\gamma T_i)}\right)^{\beta}, \tag{12}$$

where a and β are constants and:

$$d_{\text{eff}} = r_1 - r_b \tag{13}$$

where r_1 is the radial distance to the bottom of the lid.

The temperature at the base of the lid can be found from the conductive temperature profile in the lid:

$$T_L = T_s + \frac{\rho H}{6k}\left(r_t^2 - r_1^2\right) + \left(\frac{F_{\text{eff}} r_1^2}{k} - \frac{\rho H r_1^3}{3k}\right) \times\left(\frac{1}{r_1} - \frac{1}{r_t}\right). \tag{14}$$

If the stagnant lid is taken to be the cold thermal boundary layer, then $T_i = T_L$. A more accurate scaling is obtained if it is recognized that the actual lid does not include the rheological boundary layer. With this correction, the interior temperature $T_i = T_L + a_{rh}\gamma^{-1}$ where a_{rh} is a numerical coefficient (Grasset and Parmentier, 1998; Reese et al., 1999).

Substituting Eq. (11) in Eq. (12) and using the definitions of d_{eff} and $Ra_{H,0}$ yields the fitting formula:

$$\frac{1/3r_1\left(1 - r_b^3/r_1^3\right)(r_1 - r_b)^{1-3\beta}(r_1 - r_b)^{5\beta}}{\left[1/6r_t^2\left(1 - 3r_b^2/r_t^2 + 2r_b^3/r_t^3\right)\right]^{1+\beta}} = a\theta^{-(1+\beta)}Ra_{H,0}^{\beta}\exp(\theta\beta/Nu). \tag{15}$$

The radial distance to the lid bottom r_1 is found from Eq. (14):

$$r_1^3 + \left[\left(Nu^{-1} - a_{rh}\theta^{-1}\right)r_t^2\left(1 - 3r_b^2/r_t^2 + 2r_b^3/r_t^3\right) - r_t^2\left(1 + 2r_b^3/r_t^3\right)\right]r_1 + 2r_b^3 = 0. \tag{16}$$

A full inversion for all three parameters yields $a = 2.8 \pm 1.7$, $\beta = 0.21 \pm 0.06$, and $a_{rh} = 6.5 \pm 4.6$. The errors were bootstrapped (Efron and Tibshirani, 1986) and correspond to a 68% confidence interval. Due to the insensitivity of the model to the value of a_{rh} the radial distribution of interior horizontally averaged velocity gradient (Fig. 2) was used to better constrain this parameter: $a_{rh} \approx 3.7 \pm 0.6$. This is similar to the definition of Davaille and Jaupart (1993) who constrain the lid thickness with the convective heat flow distribution. The discrepancy between this value for a_{rh} and previous estimates (Davaille and Jaupart, 1993, 1994; Grasset and Parmentier, 1998; Reese et al., 1999) may be due to resolution error, definition of the stagnant lid, or might represent a fundamental difference between convection in Cartesian and spherical shell geometries. In any case, variations in a_{rh} do not have a large effect on the heat flux. The value obtained for β is very close to the theoretical value of $1/5$ (Morris and Canright, 1984; Fowler, 1985). Fixing $\beta = 0.2$ and $a_{rh} = 3.7$ yields $a = 2.51 \pm 0.05$ with 95% confidence interval error bounds (Fig. 3).

These values for the model parameters complete the solution for the mantle heat flux in the low Nusselt number regime. To compare the results with two-dimensional square box calculations, we define

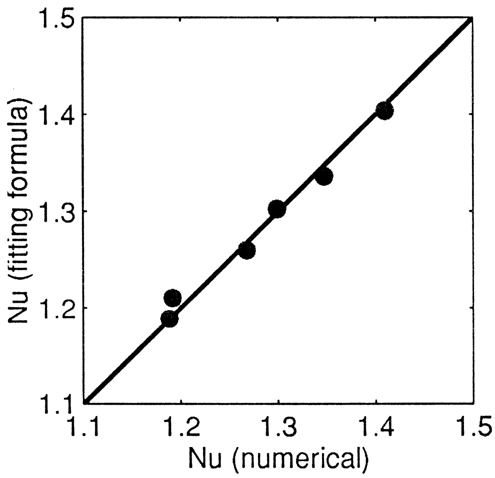

Fig. 3. The results of the fit for the Nusselt number. The values calculated using the fitting formula are plotted vs. the numerical data.

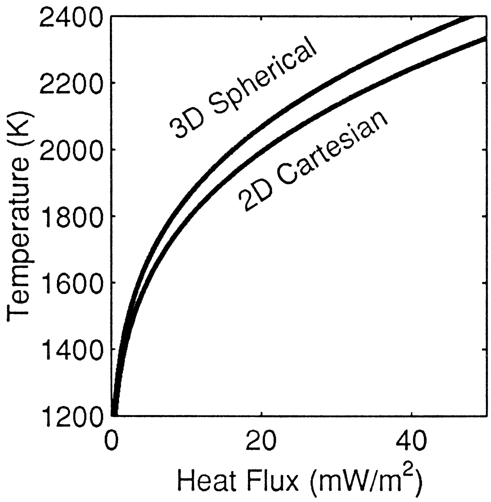

Fig. 4. The interior temperature of Mars as a function of the surface heat flux for two-dimensional (Moresi and Solomatov, 1995) and spherical shell (Eq. (17)) parameterizations. The rheological parameters correspond to dry olivine in diffusion creep (see Table 2).

a Nusselt number in terms of the total energy production in the shell:

$$Nu' = \frac{Fd}{k(T_i - T_s)} = \frac{\rho H d r_t (1 - r_b^3/r_t^3)}{3k(T_i - T_s)}. \quad (17)$$

In the plane layer limit $(r_t - r_b)/r_b \to 0$, Eq. (17) is equivalent to previous definitions (Grasset and Parmentier, 1998; Reese et al., 1999) and is physically similar for all heating modes. In the asymptotic limit ($d_{eff} \to d$ and $F_{eff} \approx F$) the best fit model parameters and Eq. (12) give:

$$Nu' \approx 2.51\theta'^{-1.2} Ra_i^{0.2}, \quad (18)$$

where θ' and Ra_i are the Frank–Kamenetskii parameter and Rayleigh number based on the internal temperature:

$$\theta' = \gamma(T_i - T_s), \quad (19)$$

$$Ra_i = \frac{\alpha\rho g (T_i - T_s) d^3}{\kappa\eta_i} \quad (20)$$

and $\eta_i = b' \exp(-\gamma T_i)$ is the internal viscosity. It is worth noting that due to limited data and low resolution we are unable to eliminate the possibility of an alternative scaling $Nu' \sim \theta'^{-1}$ (Fowler, 1985). In fact, it turns out that two-dimensional square box calculations (Moresi and Solomatov, 1995) give:

$$Nu' \approx 1.89\theta'^{-1.02} Ra_i^{0.2}. \quad (21)$$

5. Discussion and conclusions

The scaling (Eq. (18)) can be dimensionalized with physical values appropriate for the terrestrial planets. Fig. 4 shows such a calculation for Mars (Table 2) with a mantle rheology corresponding to dry olivine in diffusion creep (Karato and Wu, 1993):

$$\eta_i = \frac{\mu}{A}\left(\frac{h}{B}\right)^m \exp\left(\frac{Q}{RT_i}\right), \quad (22)$$

where A is the preexponential factor, μ is the shear modulus, h is the grain size, B is the Burgers vector, m is the grain size exponent and $Q = E + pV$ where E is the activation energy, V is the activation volume, and p is the pressure.

It should be emphasized that in the stagnant lid regime the asymptotic scaling relationship between the heat flux and interior temperature is approximately independent of heating mode. This is evident from scaling theory (Solomatov, 1995) and from comparison of two-dimensional numerical results for stagnant lid convection with bottom (Dumoulin et al., 1999) and internal heating (Grasset and Parmen-

Table 2
Mars parameters

Parameter	Notation	Value
Layer depth	d	1663 km
Thermal conductivity	k	4 W/m K
Thermal expansivity	α	2×10^{-5} K^{-1}
Density	ρ	3470 kg m^{-3}
Gravity	g	3.7 m s^{-2}
Thermal diffusivity	κ	9.6×10^{-7} m^2 s^{-1}
Surface temperature	T_s	220 K
Preexponential factor	A	8.7×10^{15} s^{-1}
Shear modulus	μ	80 GPa
Grain size	h	1.0 mm
Burgers vector	B	0.5 nm
Grain size exponent	m	2.5
Activation energy	E[a]	300 kJ mol^{-1}
Activation volume	V	6 kJ mol^{-1}

[a]Enthalpy is constant and calculated at 3.1 GPa.

tier, 1998; Reese et al., 1999). The only dynamical difference is the existence of a thin bottom thermal boundary layer in the case of bottom heating and a somewhat different flow pattern.

The results of this study suggest that the stagnant lid heat flux scaling is also similar for spherical shell and two-dimensional Cartesian geometries. The Nu–Ra relation (Eq. (18)) agrees well with scalings based on asymptotic boundary layer theories (Morris and Canright, 1984; Fowler, 1985) and bottom heated two-dimensional numerical simulations (Moresi and Solomatov, 1995). In particular, it predicts almost the same mantle temperatures and supports the conclusion that the efficiency of heat transfer is very low in the stagnant lid regime (Fig. 4). In the absence of lid remobilization, this would result in widespread melting of planetary interiors, extensive volcanism and differentiation of radiogenic elements (Reese et al., 1998, 1999).

It is worth noting that two-dimensional numerical experiments indicate that the time dependent, stagnant lid convective heat flux scales differently from that of the steady state regime (Dumoulin et al., 1999; Reese et al., 1999). The good agreement between two and three-dimensional results for the steady state cases considered in this work suggests that scalings based on two-dimensional time dependent stagnant lid calculations may also apply in spherical geometry. This can be tested by future studies.

Acknowledgements

The authors wish to thank P. Tackley and U. Christensen for constructive reviews which greatly benefited the paper. This work was supported by the NASA grant NAG5-6897, Alfred P. Sloan Foundation and New Mexico Universities Collaborative Research Program.

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